APPROXIMATE MODEL OF MELTING OF SEMITRANSPARENT BODIES IN PULSE IRRADIATION

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An approximate model of melting of semitransparent media with formation of an isothermal transition zone is constructed which makes it possible to establish possible regimes of melting and to analytically calculate the dynamics of the process under the action of pulse radiation flux.

By semitransparent (or, in other words, partially transparent) materials are meant substances which, in certain regions of the spectrum, transmit thermal radiation incident on them for a considerable distance into the depth of the specimen [1]. Practically all dielectrics in various structural states are among these materials, including ice and snow, which are widespread in nature. Under radiation flux incident on the outside of the specimen of some medium, the latter can attain the melting temperature and then undergo a phase transition. If the medium is nontransparent in relation to radiation (metals are an example practically over the entire range of the thermal radiation spectrum), then its melting starts from the surface: the melting front divides the melt and the solid phase. A mathematical description of this process is given by different variants of the classical Stefan problem, to which much research is devoted (see, for example, [2-6]).

To describe a phase transition in semitransparent media under thermal radiation, including the intrinsic radiation of the substance, use was also made of the classical Stefan problem [1, 7, 8]. The employed methods for specifying the boundary condition on a moving boundary are surveyed in [9]. However, as a theoretical analysis [10] and numerical calculations [8] show, under certain conditions paradoxical situations occur, which invoke no explanation within the framework of the classical Stefan problem — the zones of superheating of the solid phase emerge. The analysis leads to the following physical picture of melting of semitransparent bodies. Having absorbed the radiation penetrating into the body and having attained the melting temperature, the volume element of the partially transparent substance is transformed into a liquid state not instantly but in some time. This time is determined by the intensity of the radiation source and by thermophysical and optical properties of the material. In the semitransparent material layer three zones are formed: a solid which has not attained the melting temperature, an intermediate isothermal zone, formed by a mixture of the melt and the solid phase with different volume content of the melt, and, finally, a zone of the totally melted material. The emergence of the intermediate zone formed in the interaction of radiation with semitransparent media was observed and described in [9], According to this physical picture a mathematical formulation of relevant problems is also bound to be changed.

Generalized statements of the problems allowing for the presence of the intermediate zone are considered in [9-18] as applied to the problems of melting and crystallization, thawing of frozen grounds, and to geophysical problems.

The present work considers in a one-dimensional statement the approximate model of melting of a semitransparent medium under pulse irradiation. By pulse radiation, according to [19], we will mean radiation emitted for a limited, fairly small time. Gas-discharge pulse lamps, strong-current carbon arcs, power incandescent lamps, and others [19] as well as luminous bodies produced in large-scale accidents and explosions [20] may serve as pulse radiation sources of moderate intensity. The operating time of the mentioned sources is as a rule of the order of 10^{-2}-10^{0} sec, the characteristic time t_0 of irradiation rise to the maximum is fractions of a second, and the maximum irradiation F_0 is of the order of 10^5-10^8 W/m^2. The model of melting a semitransparent medium discussed below takes account of the pulse character of irradiation and enables us to analytically calculate the dynamics of the process of melting and the limiting thickness of the melted layer.

1. The basis for the approximate model of melting of semitransparent media is formed by the following assumptions. It is believed that in melting there occurs no variation in the density of the material so as not to take into account convective streams arising with different densities; the refractive indexes of the solid and the melt are similar and there is no internal reflectionat the solid–melt interfaces. The approximation of a gray substance is taken. It is believed that the intermediate iso-

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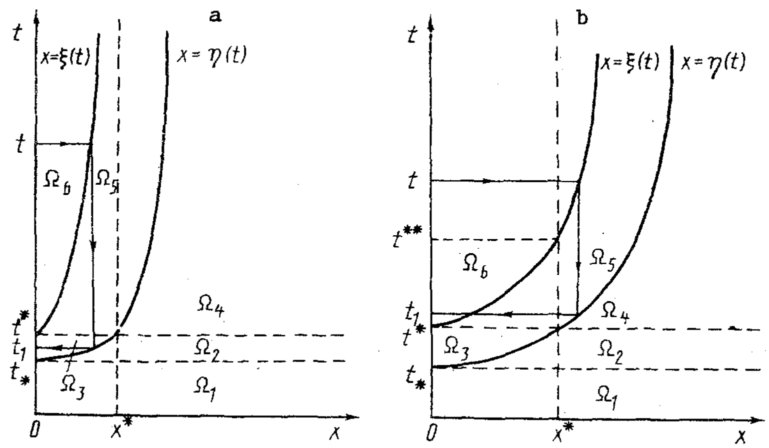

Fig. 1. Layout of two melting fronts on the plane (x, t).

thermal zone formed by a mixture of the melt and the solid phase has an extinction coefficient, independent of the melt fraction. In connection with the given assumption we will point out that the physical properties of the transient zone formed in melting or crystallization of semitransparent bodies have hardly been investigated to date [9, 11]. The intrinsic radiation of a medium is assumed to be negligible in comparison with the absorbed radiation (approximation of a cold substance). To describe the extinction of the radiation flux in the bulk of the semitransparent substance we employ Bouguer's law with an effective extinction coefficient independent of the medium temperature. Finally, the pulse character of irradiation with rather high intensity assumes rapid heating of the medium when heat conduction is of negligible importance.

We will consider in more detail the last assumption, which is of substantial importance in constructing the approximate model of melting. Assume that there is given a half-space $x \geq 0$, filled with a partially transparent substance with the extinction coefficient a_1, the reflectivity R_1, and the initial temperature $T_0 < T_*$. For $t \geq 0$ the half-space is externally irradiated by the pulse radiation flux described by the function $F(t) = F_0 f(t/t_0)$. Using the dimensionless variables $u = (T - T_0)/\Delta T$, $\theta = t/t_0$, and $y = a_1 x$ the mathematical formulation of the problem of heating up the half-space with constant thermophysical characteristics until its elements attain the melting temperature has the form

$$\frac{\partial u}{\partial \theta} = b\frac{\partial^2 u}{\partial y^2} + d_1 \exp(-y) f(\theta), \tag{1}$$

$$y > 0,\ 0 < \theta < \theta_m, \tag{2}$$

$$u(y,\ 0) = 1,\ y \geqslant 0,$$
$$\frac{\partial u(0,\ \theta)}{\partial y} = 0,\ \theta \geqslant 0. \tag{3}$$

Here

$$b = \frac{\varkappa a_1^2 t_0}{\rho c};\quad d_1 = \frac{a_1(1 - R_1) F_0 t_0}{\rho c \Delta T};$$

θ_m is the dimensionless time of attaining the melting temperature. The condition (3) characterizes the absence of heat exchange between the half-space and the ambient medium. In many cases the dimensionless coefficient b is a small quantity. For example, for ice $\varkappa = 2.23$ W/(m·K), $a_1 = 5$ m^{-1} [21], $\rho = 917$ kg/m^3, $c = 2.1 \times 10^3$ J/(kg·K). Assuming $t_0 = 1$ sec, we obtain $b = 2.9 \times 10^{-5}$. In this connection along with problem (1)-(3) we will deal with the following Cauchy problem, formally corresponding to the case $b = 0$:

$$\frac{\partial U}{\partial \theta} = d_1 \exp(-y) f(\theta),\ \theta \geqslant 0, \tag{4}$$

$$U(y,\ 0) = 1,\ y \geqslant 0. \tag{5}$$

Using estimates analogous to those performed in [22] it can be shown that for all $y \geq 0$, $0 < \theta < \theta_m$ there exists an equality

$$\lim_{b \to 0} u(y, \theta) = U(y, \theta), \tag{6}$$

which indicates that for fairly small values of the parameter b in practical calculations, instead of (1)-(3), we can use the solution of the problem (4)-(5) ignoring heat conduction. This circumstance substantially simplifies the analysis and is used below to construct the approximate model of melting of semitransparent media.

2. We go to a mathematical description of the approximate model. Let the half-space $x \geq 0$ be partially filled with a semitransparent medium with the extinction coefficient a_1, the reflectivity R_1, and the initial temperature $T_0 < T_*$. For $t \geq 0$, the half-space surface is irradiated by the pulse radiation flux described by the function $F(t) = F_0 f(t/t_0)$. The work [19] proposes dependences for F(t), formed by conjugating various analytical expressions. Below we will consider the function F(t) to be determined on the interval $[0, +\infty)$, nonnegative, piecewise continuous, and fulfulling the condition

$$\int_0^\infty F(t)\,dt < +\infty.$$

When needed F(t) may be considered equal to zero with $t > t_s$, where t_s is the operating time of the radiation source.

Figure 1 shows the plane of variables (x, t); plotted are the boundaries of a two-phase intermediate zone (the leading edge $x = \xi(t)$ and the trailing edge $x = \eta(t)$). Here too the regions Ω_i ($i = 1, 2, \ldots, 6$) are set aside. In each of them we write an equation for the temperature of medium T or for the melt fraction z ($0 \leq z \leq 1$) according to the assumptions formulated above. We agree to provide the quantities T and z with the index fitting the considered region Ω_i. We have the following problems: in the region Ω_1

$$\rho c \frac{\partial T_1}{\partial t} = a_1 (1 - R_1) \exp(-a_1 x) F(t), \; 0 < t < t_*, \; T(x, 0) = T_0; \tag{7}$$

in the region Ω_2

$$\rho c \frac{\partial T_2}{\partial t} = a_1 (1 - R_2) \exp[-a_2 \xi(t) - a_1 (x - \xi(t))] F(t), \; t_* < t < t^*,$$
$$T_2(x, t_*) = T_1(x, t_*); \tag{8}$$

in the region Ω_3

$$\rho L \frac{\partial z_3}{\partial t} = a_2 (1 - R_2) \exp(-a_2 x) F(t), \; t_* < t < t^*,$$
$$z_3(x, t(x)) = 0. \tag{9}$$

Here $t = t(x)$ is the function inverse to $x = \xi(t)$, and L is the melting specific heat. Let us introduce the notation $x^* = \xi(t^*)$. In the region Ω_4 we have

$$\rho c \frac{\partial T_4}{\partial t} = a_1 (1 - R_3) \exp[-a_3 \eta(t) - a_2 (\xi(t) - \eta(t)) -$$
$$- a_1 (x - \xi(t))] F(t), \; t > t^*,$$
$$T_4(x, t^*) = T_2(x, t^*), \; x \geqslant x^*. \tag{10}$$

In the region Ω_5

$$\rho L \frac{\partial z_5}{\partial t} = a_2 (1 - R_3) \exp[-a_3 \eta(t) - a_2 (x - \eta(t))] F(t), \; t > t^*,$$
$$z_5(x, t^*) = z_3(x, t^*), \; 0 \leqslant x < x^*,$$
$$z_5(x, t(x)) = 0, \; x \geqslant x^*. \tag{11}$$

The problem for the temperature T_6 in the region Ω_6 in the given work is of auxiliary character and therefore is not dealt with. Equations (9) and (11) for the melt fraction z(x, t) in the intermediate zone follow from consideration of the energy balance for the element of a melting substance [9].

To complete the statement of the problems (7)-(11), additional conditions on the moving boundaries $x = \xi(t)$ and $x = \eta(t)$ should be indicated. In the general case these conditions have the form [9] ($\varkappa_l$ is the thermal conductivity of the melt)

$$T_i(\xi(t)+0,\ t) = T_*,\ \frac{\partial T_i(\xi(t)+0,\ t)}{\partial x} = 0,\ i = 2,4; \tag{12}$$

$$T_6(\eta(t)-0,\ t) = T_*,\ -\varkappa_l \frac{\partial T_6(\eta(t)-0,\ t)}{\partial x} = [1 - z_5(\eta(t)+0,\ t)]\rho L \frac{d\eta}{dt}. \tag{13}$$

In the approximate model studied, according to the assumption of negligible influence of heat conduction and in view of the first order of Eqs. (7)-(11), we use below the following closing relations, resulting from Eqs. (12) and (13):

$$T_i(\xi(t),\ t) = T_*,\ i = 2,\ 4, \tag{14}$$

$$z_5(\eta(t),\ t) = 1. \tag{15}$$

In expressions (7)-(11) the following values are to be determined: t_* is the instant in which the surface of the semitransparent medium attains the melting temperature; t^* is the instant where the rear boundary of the intermediate two-phase zone forms; T_1, T_2, and T_4 are the temperature fields in the regions Ω_1, Ω_2, and Ω_4, respectively; z_3 and z_5 are the distribution of the melt fraction in the regions Ω_3 and Ω_5; $x = \xi(t)$ and $x = \eta(t)$ are the leading and trailing edges of melting, limiting the intermediate isothermal zone. Besides, different regimes of heating a medium are realizable, which are characterized by the absence of one or both edges of melting. This is to be determined in the process of solution. The remaining values in (7)-(11) are considered prescribed.

Let us take a closer look at the value of reflectivity of the two-phase zone. With $t_* \le t \le t^*$ the reflectivity R_2 depends, in general, on the value of z. Thus, in observations of thawing snow under solar radiation it is established [21] that as the water fraction on the day surface of the ground increases, the reflectivity of the snow monotonically decreases. Taken below is the linear dependence of the reflectivity of the melting medium on the melt fraction on the surface:

$$R_2 = R_2(t) = R_3 z(\ ,\ t) + [1 - z(0,\ t)]R_1. \tag{16}$$

The explicit form of the dependence of R_2 on time with $t_* \le t \le t^*$ is determined below.

3. We present a procedure for constructing the solution of the problems (7)-(12), (14), and (15). First of all we note that if the dependences $x = \xi(t)$ and $x = \eta(t)$ are established then $T_i(x, t)$ and $z_i(x, t)$ can be found from (7)-(11) by simple integration. Therefore, most attention will be concentrated below on finding the leading and trailing edges of melting and determining the criteria of their formation. Using (7) we determine the temperature distribution in the solid medium up to the moment the melting starts:

$$T_1(x,\ t) = T_0 + \frac{a_1(1-R_1)}{\rho c} \exp(-a_1 x)\Phi(t). \tag{17}$$

Here and below use is made of the function

$$\Phi(t) = \int_0^t F(\tau)\, d\tau.$$

We will introduce the dimensionless quantity

$$K_1 = \frac{a_1 (1 - R_1) \Phi(\infty)}{\rho c \Delta T}. \tag{18}$$

The criterion of melting of the semitransparent medium and formation of the leading edge $x = \xi(t)$ is specified by the inequality

$$K_1 \geqslant 1, \tag{19}$$

which follows from (17) when $x = 0$ and corresponds to the condition

$$\lim_{t\to\infty} T_1(0,\ t) \geqslant T_*.$$

When inequality (19) is met, the instant t_* is determined from the equation

$$\Phi(t_*) = \frac{\rho c \Delta T}{a_1 (1 - R_1)},$$

whose solution exists and is unique by virtue of continuity and monotony of the function $\Phi(t)$. With $K_1 < 1$ melting does not set in.

Now we consider problem (9). It follows from it that z_3 acquires its maximum value when $x = 0$. Assuming in (9) that $x = 0$ and using (16), we obtain an expression for $z_3(0, t)$

$$z_3(0,\ t) = \frac{1 - R_1}{R_1 - R_3}\left\{\exp\left[\frac{a_2 (R_1 - R_3)}{\rho L}(\Phi(t) - \Phi(t_*))\right] - 1\right\}.$$

From this we will determine the functional dependence of the reflectivity R_2 on time

$$R_2(t) = 1 - (1 - R_1)\exp\left[\frac{a_2 (R_1 - R_3)}{\rho L}(\Phi(t) - \Phi(t_*))\right].$$

The fulfillment of the inequality

$$\lim_{t\to\infty} z_3(0,\ t) \geqslant 1 \tag{20}$$

signifies the formation of the trailing edge $x = \eta(t)$. Relation (20) may be written in the equivalent form

$$K_2 \geqslant 1, \tag{21}$$

where

$$K_2 = \frac{a_2 (R_1 - R_3)[\Phi(\infty) - \Phi(t_*)]}{\rho L \ln[(1 - R_3)/(1 - R_1)]}.$$

Inequality (21) represents the criterion of formation of the trailing edge of melting for the semitransparent medium. With its fulfillment the time t^* is determined by the equation

$$\Phi(t^*) = \Phi(t_*) + \frac{\rho L}{a_2 (R_1 - R_3)} \ln \frac{1 - R_3}{1 - R_1},$$

whose solution exists and is unique.

Let us proceed to finding the functions $\xi(t)$ and $\eta(t)$. The inequalities $K_1 > 1$ and $K_2 > 1$ will be considered fulfilled and the values of t_* and t^* found. With the aim of determining the front $x = \xi(t)$ for $t_* \le t \le t^*$ we deal with problem (8). Taking into account that $T_1(x, t) = T_0 + \Delta T\exp(-a_1x)$, we integrate Eq. (8) and employ condition (14). We arrive at the integral equation for determining $\xi(t)$:

$$\exp[a_1\xi(t)] = 1 + \frac{a_1}{\rho c\Delta T}\int_{t^*}^{t}[1 - R_2(\tau)]\exp[(a_1 - a_2)\xi(\tau)]F(\tau)\,d\tau, \tag{22}$$

whose solution has the form

$$\xi(t) = \frac{1}{a_2}\ln\left\{1 + \frac{a_2}{\rho c\Delta T}\int_{t_*}^{t}[1 - R_2(\tau)]F(\tau)\,d\tau\right\}. \tag{23}$$

Two variants are further possible for the layout of the lines $\xi(t)$ and $\eta(t)$ on the plane (x, t). In the first case the front $x = \eta(t)$ does not cross the straight line $x = x^* = \xi(t^*)$ (see Fig. 1a). In the second case $\lim_{t\to\infty} \eta(t) > x^*$ (see Fig. 1b). We begin by considering the first possibility. To find $\eta(t)$ we use problem (11), where $z_3(x, t^*)$ is determined from (9)

$$z_3(x, t^*) = \frac{a_2\exp(-a_2x)}{\rho L}\int_{t_1}^{t^*}[1 - R_2(\tau)]F(\tau)\,d\tau. \tag{24}$$

Here t_1 $(t^* \le t_1 \le t^*)$ is the instant satisfying the condition $\xi(t_1) = \eta(t)$ (see Fig. 1a). Integrating (11) in view of (24) and using the condition (15) on the trailing edge, we arrive at the integral equation

$$\exp[a_2\eta(t)] = \frac{a_2}{\rho L}\int_{t_1}^{t^*}[1 - R_2(\tau)]F(\tau)\,d\tau + \frac{a_2(1 - R_3)}{\rho L}\int_{t^*}^{t}\exp[(a_2 - a_3)\eta(\tau)]F(\tau)\,d\tau.$$

In view of the equality $\xi(t_1) = \eta(t)$ and formula (23) the given equation can be transformed to the form

$$\exp[a_2\eta(t)] = 1 + \frac{a_2(1 - R_3)}{\rho(L + c\Delta T)}\int_{t^*}^{t}\exp[(a_2 - a_3)\eta(\tau)]F(\tau)\,d\tau. \tag{25}$$

Differentiating the both sides of (25) and using the relation $\eta(t^*) = 0$, we find

$$\eta(t) = \frac{1}{a_3}\ln\left\{1 + \frac{a_3(1 - R_3)}{\rho(L + c\Delta T)}[\Phi(t) - \Phi(t^*)]\right\}. \tag{26}$$

The function $x = \xi(t)$ for $t \ge t^*$ remains to be determined. For this purpose we make use of problem (10) together with condition (14). We obtain the integral equation,

$$\exp[a_1\xi(t)] = 1 + \frac{a_1}{\rho c\Delta T}\int_{t_*}^{t^*}[1 - R_2(\tau)]\exp[(a_1 - a_2)\xi(\tau)]F(\tau)\,d\tau + \frac{a_1(1 - R_3)}{\rho c\Delta T}\int_{t^*}^{t}\exp[(a_2 - a_3)\eta(\tau) + (a_1 - a_2)\xi(\tau)]F(\tau)\,d\tau,$$

in which the above-found functions $\xi(t)$ ($t_x \le t \le t^*$, formula (23)) and $\eta(t)$ ($t \ge t^*$, formula (26)) appear. After transformations, with the use of these dependences, we have

$$\exp[a_2\xi(t)] = \frac{c\Delta T + L}{c\Delta T} + \frac{a_2(1 - R_3)}{\rho c\Delta T}\int_{t^*}^{t}\exp[(a_2 - a_3)\eta(\tau)]F(\tau)\,d\tau.$$

From this equation in view of (25) we can obtain

$$\xi(t) = \eta(t) + \frac{1}{a_2}\ln\left(1 + \frac{L}{c\Delta T}\right). \tag{27}$$

The expression for $\eta(t)$ is given by formula (26).

We now turn to the second possibility of the layout of the two-phase zone boundaries (see Fig. 1b). The leading edge of melting for $t_* \leq t \leq t^*$ is as previously described by formula (2). Let us denote by t^{**} the instant in which the equality $\eta(t^{**}) = \xi(t^*)$ is satisfied (see Fig. 1b). Then for $t^* \leq t \leq t^{**}$ the rear boundary of the two-phase zone $x = \xi(t)$ is determined similarly to the previous case and is specified by formula (26). Taking advantage of the problems (9) and (11) as well as of conditions (14), (15), we arrive at the following system of integral equations for determining the functions $\xi(t)$ with $t \geq t^*$ and $\eta(t)$ with $t \geq t^{**}$:

$$\exp[a_1\xi(t)] = 1 + \frac{a_1}{\rho c\Delta T}\int_{t_*}^{t^*}[1 - R_2(\tau)]\exp[(a_1 - a_2)\xi(\tau)]F(\tau)\,d\tau +$$
$$+ \frac{a_1(1-R_3)}{\rho c\Delta T}\int_{t_*}^{t}\exp[(a_2 - a_3)\eta(\tau) + (a_1 - a_2)\xi(\tau)]F(\tau)\,d\tau, \quad t \geqslant t^*, \tag{28}$$
$$\exp[a_2\eta(t)] = \frac{a_2(1-R_3)}{\rho L}\int_{t_1}^{t}\exp[(a_2 - a_3)\eta(\tau)]F(\tau)\,d\tau, \quad t \geqslant t^{**},$$

where t_1 is such that $\xi(t_1) = \eta(t)$ (see Fig. 1b). Omitting intermediate calculations, we write the solution of the system (28)

$$\eta(t) = \frac{1}{a_3}\ln\left\{1 + \frac{a_3(1-R_3)}{\rho(L + c\Delta T)}[\Phi(t) - \Phi(t^*)]\right\},$$
$$\xi(t) = \eta(t) + \frac{1}{a_2}\ln\left(1 + \frac{L}{c\Delta T}\right). \tag{29}$$

As is evident, the functional dependences (29) coincide with those established above for the case shown in Fig. 1a. From (29) it follows that within the framework of the model in question the width h of the two-phase transion zone is a constant quantity independent of time. As the coefficient a_2 increases, i.e., as the opacity of the medium increases, h tends to zero, which is consistent with conclusions drawn in [9]. The limiting thickness of the melted substance is determined from (29) as $t \to \infty$; this value is finite according to the assumption of the pulse character of radiation.

On finding the explicit expressions (29) for $\xi(t)$ and $\eta(t)$ the determination of the dependences $T_i(x, t)$ and $z_i(x, t)$ reduces to quadratures and can be performed by simple integration of Eqs. (7)-(11). We note that as t increases, the growth of the error of the approximate solution is to be expected, since, as the pulse radiation source attenuates, heat conduction may be impossible to ignore. In this case the model is complicated by introducing the corresponding terms into Eqs. (7)-(11) in view of the additional conditions (12), (13).

4. Thus, constructed above is the approximate analytical model of melting of semitransparent media, externally irradiated by pulse radiation flux. The basis of the model is formed by the assumption of the dominant character of heating the medium through the absorption of radiation in comparison with heat conduction. The model establishes easily checkable criteria of formation of the leading and trailing edges of melting, makes it possible to analytically, without resorting to cumbersome calculations, determine the dynamics of the process of melting, to find the temperature field and the melt fraction distribution in a two-phase isothermal zone, and to estimate the limiting thickness for the melted material. The established dependences may be used in practice to promptly estimate the impact of exposure to pulse irradiation of partially transparent materials and also to serve as a test in numerical calculations by complicated models. The proposed model may be complicated by taking into account the spectral dependence of the extinction coefficient and the reflectivity of the substance, the nonuniform initial temperature, and the dependence of a_2 on the melt fraction z. In this case, numerical calculations will be required; however, a qualitative picture of phenomena will not change.

NOTATION

x, coordinate; t, time; t_s, operating time of the radiation source; R_1, R_2, R_3, reflectivity of the medium, the intermediate zone, and the melt; a_1, a_2, a_3, extinction coefficients of the solid phase, the intermediate zone, and the melt; T_0, initial temperature; T_*, melting temperature; U, u, θ, y, dimensionless parameters; $\varkappa$, thermal conductivity; ρ, density; c, specific heat capacity; L, specific heat of melting; F, intensity of the radiation flux; f, dimensionless intensity of the radiation flux; ξ, η, leading and trailing edges of melting; z, melt fraction; Ω, region on the plane (x, t); K_1, K_2, dimensionless criteria of the regimes of melting; Φ, integral of the radiation flux; $\Delta T = T_* - T_0$; h, intermediate zone thickness. Indexes: *l* corresponds to the liquid phase.

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